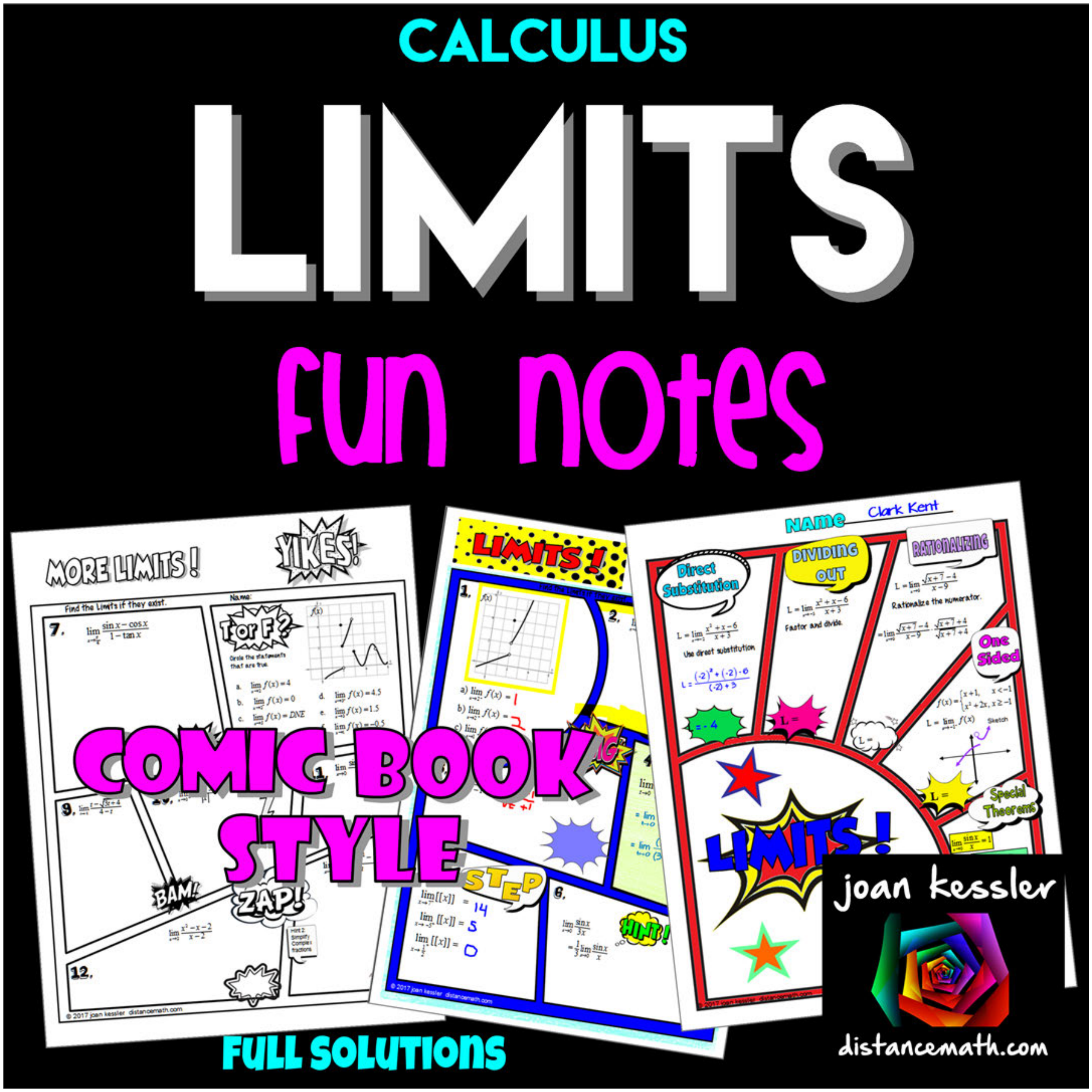
CALCULUS
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Name ______________________

Direct Substitution

$$L = \lim_{x \to -2} \frac{x^2 + x - 6}{x + 3}$$

Use direct substitution

L =

DIVIDING OUT

$$L = \lim_{x \to -3} \frac{x^2 + x - 6}{x + 3}$$

Factor and divide.

L =

RATIONALIZING

$$L = \lim_{x \to 9} \frac{\sqrt{x+7} - 4}{x - 9}$$

Rationalize the numerator.

$$= \lim_{x \to 9} \frac{\sqrt{x+7} - 4}{x - 9} \cdot \frac{\sqrt{x+7} + 4}{\sqrt{x+7} + 4}$$

L =

One Sided

$$f(x) = \begin{cases} x + 1, & x < -1 \\ x^2 + 2x, & x \geq -1 \end{cases}$$

$$L = \lim_{x \to -1^-} f(x)$$

Sketch

L =

LIMITS!

Special Theorems

$$\boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1}$$

$$\boxed{\lim_{x \to 0} \frac{1 - \cos x}{x} = 0}$$

$$\boxed{\lim_{x \to 0} \frac{\tan x}{x} = 1}$$

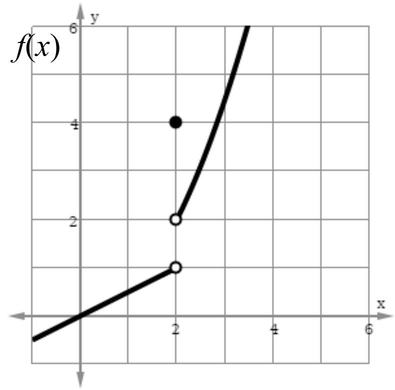
$$\boxed{\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e}$$

These theorems and can help in certain problems.

LIMITS!

Find the Limits if they exist.

1.



a) $\lim\limits_{x\to 2^-} f(x) =$

b) $\lim\limits_{x\to 2^+} f(x) =$

c) $\lim\limits_{x\to 1} f(x) =$

2. $\lim\limits_{x\to -2} (x+3)^2\sqrt{4x^2-8}$

OMG

3. $\lim\limits_{t\to 1} \dfrac{\sqrt{t}-1}{t-1}$

4. $\lim\limits_{t\to 0} \dfrac{2t^3+3t^2}{3t^4-2t^2}$

5. STEP

$\lim\limits_{x\to 7^-} [[x]] =$

$\lim\limits_{x\to -5^+} [[x]] =$

$\lim\limits_{x\to \frac{1}{2}^+} [[x]] =$

6. HINT!

$\lim\limits_{x\to 0} \dfrac{\sin x}{3x}$

$= \dfrac{1}{3}\lim\limits_{x\to 0} \dfrac{\sin x}{x}$

MORE LIMITS !

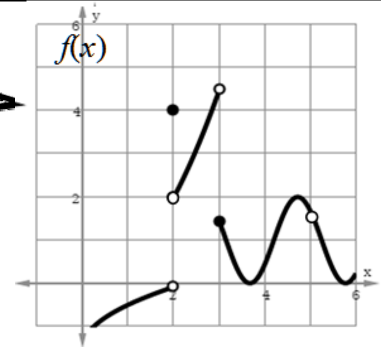
YIKES!

Find the Limits if they exist.

Name:

7. $$\lim_{x\to\frac{\pi}{4}} \frac{\sin x - \cos x}{1-\tan x}$$

8. T or F ?



Circle the statements that are true.

a. $\lim_{x\to 2} f(x) = 4$

b. $\lim_{x\to 2^-} f(x) = 0$

c. $\lim_{x\to 5} f(x) = DNE$

d. $\lim_{x\to 3^+} f(x) = 4.5$

e. $\lim_{x\to 3} f(x) = 1.5$

f. $\lim_{x\to 1} f(x) = -0.5$

9. $$\lim_{t\to 4} \frac{t-\sqrt{3t+4}}{4-t}$$

10. $$\lim_{x\to 0^-} \frac{x^2-3x}{|x|}$$

11. $$\lim_{x\to 0} \frac{\sin 2x}{x}$$

BAM!

ZAP!

12. $$\lim_{x\to 2} \frac{x^2-x-2}{x-2}$$

13. $$\lim_{x\to 0} \frac{(2+h)^{-1}-2^{-1}}{h}$$

Hint 1: $x^{-1} = \frac{1}{x}$

Hint 2: Simplify Complex fractions

Name **KEY**

LIMITS!

Direct Substitution

$$L = \lim_{x \to -2} \frac{x^2 + x - 6}{x + 3}$$

Use direct substitution

$$L = \frac{(-2)^2 + (-2) - 6}{(-2) + 3}$$

L = - 4

DIVIDING OUT

$$L = \lim_{x \to -3} \frac{x^2 + x - 6}{x + 3}$$

Factor and divide

$$L = \lim_{x \to -3} \frac{(x+3)(x-2)}{(x+3)}$$

L = **-5**

RATIONALIZING

$$L = \lim_{x \to 9} \frac{\sqrt{x+7} - 4}{x - 9}$$

Rationalize the numerator

$$= \lim_{x \to 9} \frac{\sqrt{x+7} - 4}{x - 9} \cdot \frac{\sqrt{x+7} + 4}{\sqrt{x+7} + 4}$$

$$= \lim_{x \to 9} \frac{x + 7 - 16}{(x-9)\left(\sqrt{x+7} + 4\right)}$$

$$\lim_{x \to 9} \frac{(x-9)}{(x-9)\left(\sqrt{x+7} + 4\right)}$$

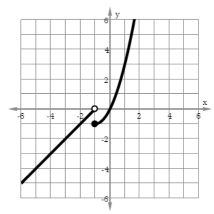
$$L = = \frac{1}{8}$$

One Sided

$$f(x) = \begin{cases} x + 1, & x < -1 \\ x^2 + 2x, & x \geq -1 \end{cases}$$

$$L = \lim_{x \to -1^-} f(x)$$

Sketch



$L = \mathbf{0}$

Special Theorems

$$\boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1}$$

$$\boxed{\lim_{x \to 0} \frac{1 - \cos x}{x} = 0}$$

$$\boxed{\lim_{x \to 0} \frac{\tan x}{x} = 1}$$

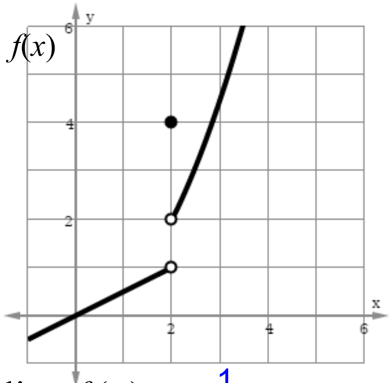
$$\boxed{\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e}$$

These theorems and can help in certain problems.

LIMITS! KEY

Find the Limits if they exist.

1.



a) $\lim_{x\to 2^-} f(x) =$ 1

b) $\lim_{x\to 2^+} f(x) =$ 2

c) $\lim_{x\to 1} f(x) =$ 1/2

d) $\lim_{x\to 2} f(x) =$ DNE

2. $\lim_{x\to -2}(x+3)^2\sqrt{4x^2-8}$

12. $\lim_{x\to -2}(x+3)^2\sqrt{4x^2-8} = 1^2\sqrt{4(-2)^2-8} = 2\sqrt{2}$

$1^2\sqrt{4(-2)^2-8}$

$2\sqrt{2}$

$2\sqrt{2}$

OMG

3. $\lim_{t\to 1}\dfrac{\sqrt{t}-1}{t-1}$

$\lim_{t\to 1}\dfrac{\sqrt{t}-1}{(\sqrt{t}+1)(\sqrt{t}-1)}$

$= \lim_{t\to 1}\dfrac{1}{\sqrt{t}+1} = \dfrac{1}{2}$

1/2

4. $\lim_{t\to 0}\dfrac{2t^3+3t^2}{3t^4-2t^2}$

$= \lim_{t\to 0}\dfrac{t^2(2t+3)}{t^2(3t^2-2)}$

$= \lim_{t\to 0}\dfrac{(2t+3)}{(3t^2-2)} - -\dfrac{3}{2}$

5. STEP

$\lim_{x\to 7^-}[[x]] =$ 6

$\lim_{x\to -5^+}[[x]] =$ -5

$\lim_{x\to \frac{1}{2}^+}[[x]] =$ 0

6. $\lim_{x\to 0}\dfrac{\sin x}{3x}$

HINT!

$= \dfrac{1}{3}\lim_{x\to 0}\dfrac{\sin x}{x}$

$= \dfrac{1}{3}\lim_{x\to 0}(1)$

$= \dfrac{1}{3}$

MORE LIMITS! KEY YIKES!

Find the Limits if they exist.

Name:

7. $\displaystyle \lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{1 - \tan x}$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{1 - \frac{\sin x}{\cos x}}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{(\sin x - \cos x)\cos x}{\cos x - \sin x}$$

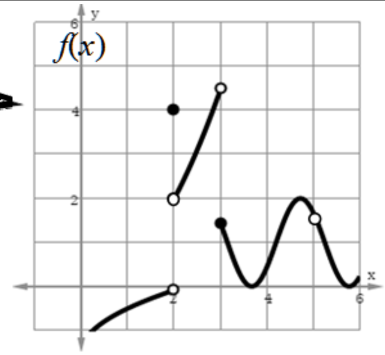
$$\lim_{x \to \frac{\pi}{4}} (-\cos x) = -\cos \frac{\pi}{4}$$

$$= -\frac{\sqrt{2}}{2}$$

$-\frac{\sqrt{2}}{2}$

8. T or F?

Circle the statements that are true.



a. $\lim_{x \to 2} f(x) = 4$

b. $\lim_{x \to 2^-} f(x) = 0$

c. $\lim_{x \to 5} f(x) = DNE$

d. $\lim_{x \to 3^+} f(x) = 4.5$

e. $\lim_{x \to 3} f(x) = 1.5$

f. $\lim_{x \to 1} f(x) = -0.5$

9. $\displaystyle \lim_{t \to 4} \frac{t - \sqrt{3t+4}}{4 - t}$

$$= \lim_{t \to 4} \frac{(t - \sqrt{3t+4})(t + \sqrt{3t+4})}{(4-t)(t + \sqrt{3t+4})}$$

$$= \lim_{t \to 4} \frac{t^2 - 3t + 4}{(4-t)(t + \sqrt{3t+4})}$$

$$= \lim_{t \to 4} \frac{(t-4)(t+1)}{(4-t)(t + \sqrt{3t+4})}$$

$$= \lim_{t \to 4} \frac{-(t+1)}{(t + \sqrt{3t+4})} = \frac{-5}{8}$$

10. $\displaystyle \lim_{x \to 0^-} \frac{x^2 - 3x}{|x|}$

$$= \lim_{x \to 0^-} \frac{x(x-3)}{-x}$$

$$= \lim_{x \to 0^-} \frac{(x-3)}{-1} = 3$$

11. $\displaystyle \lim_{x \to 0} \frac{\sin 2x}{x}$

$$= \lim_{x \to 0} \left(\frac{\sin 2x}{2x} \cdot \frac{2}{1} \right)$$

$$= 2 \lim_{x \to 0} \left(\frac{\sin 2x}{2x} \right)$$

$$= 2(1) = 2$$

BAM!

ZAP!

12. $\displaystyle \lim_{x \to 2} \frac{x^2 - x - 2}{x - 2}$

$$= \lim_{x \to 2} \frac{(x-2)(x+1)}{x-2}$$

$$= \lim_{x \to 2} (x+1) = 3$$

3

13. $\displaystyle \lim_{h \to 0} \frac{(2+h)^{-1} - 2^{-1}}{h}$

Hint 1: $x^{-1} = \frac{1}{x}$

$$\lim_{x \to 0} \frac{\frac{1}{(2+h)} - \frac{1}{2}}{h}$$

Hint 2: Simplify Complex fractions

$$\lim_{x \to 0} \frac{2 - (2+h)}{(2+h)2h}$$

$$\lim_{x \to 0} \frac{-h}{(2+h)2h} = \frac{-1}{4}$$