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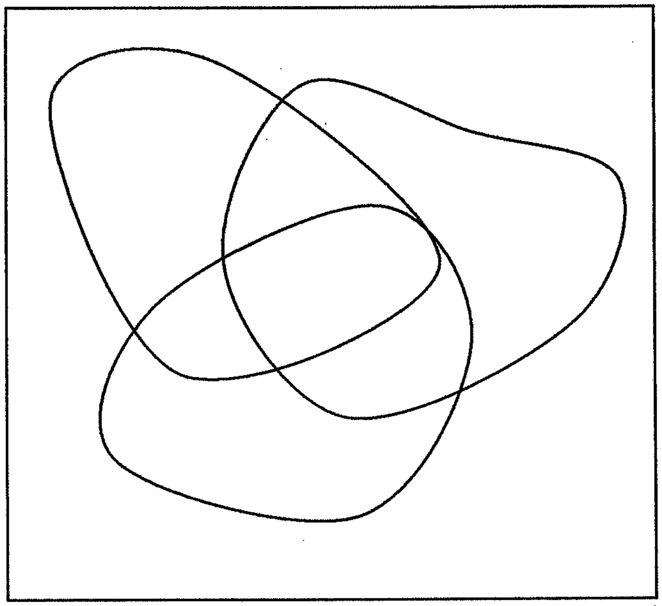

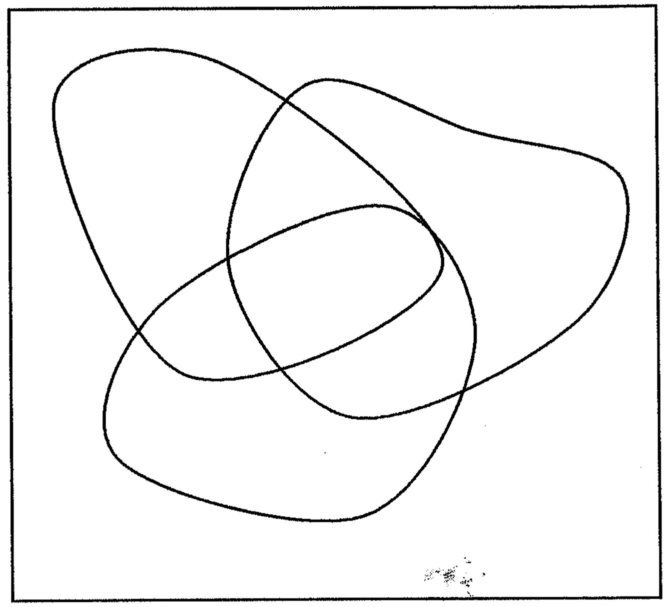

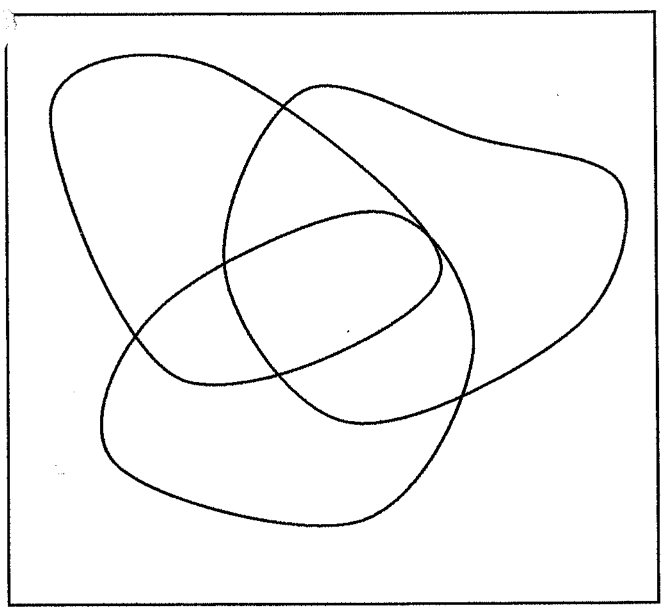

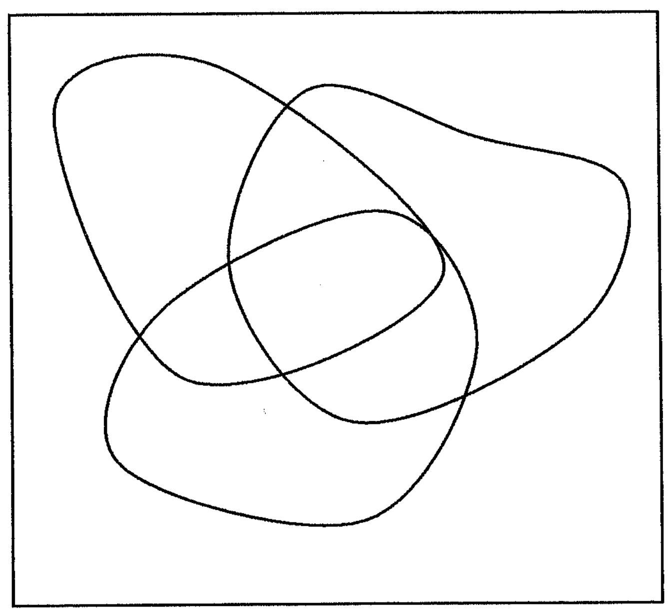

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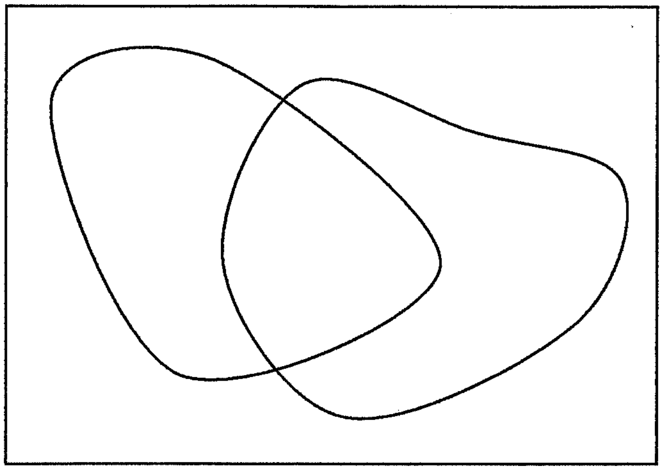
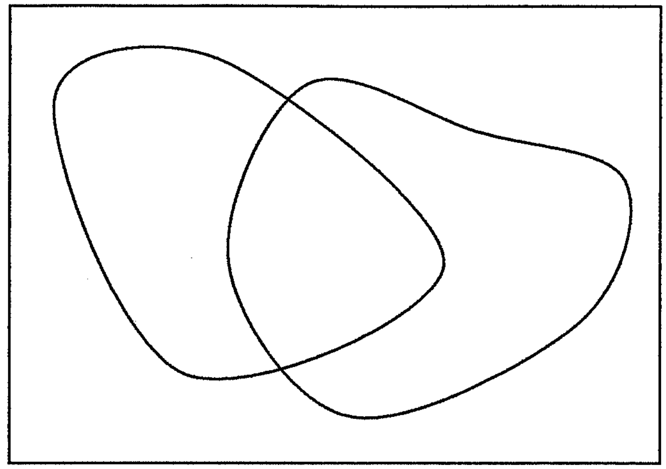
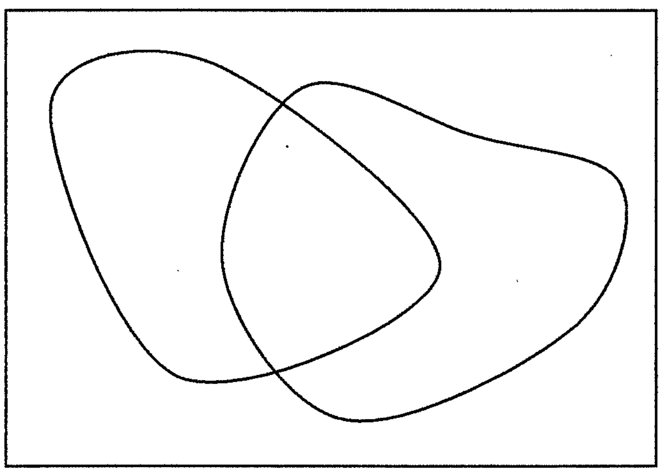
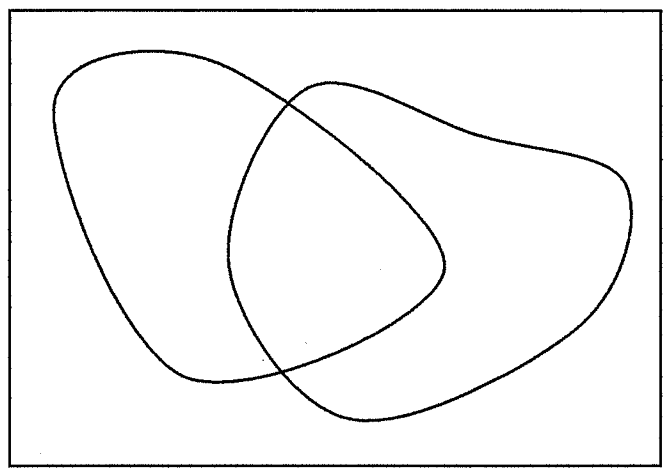
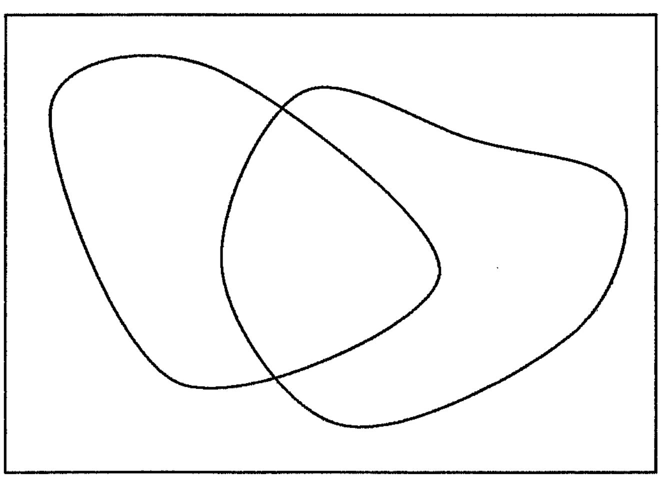
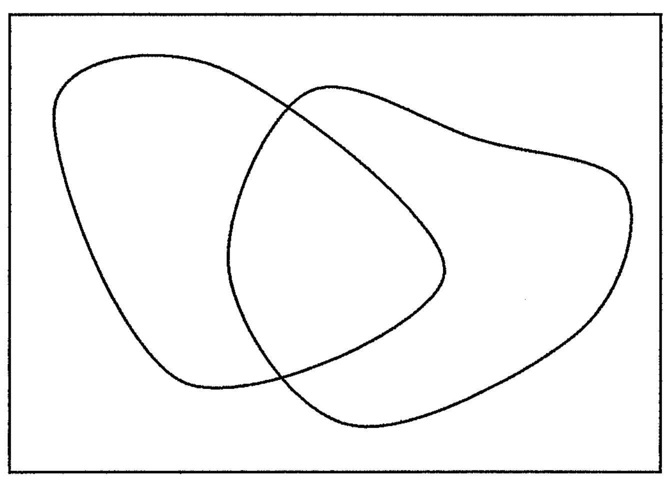












Randomness & Probability Models:

Behavior is random if, while individual outcomes are uncertain, for a large number of repetitions, outcomes are regularly distributed.

Example: If I roll a die once, I can't predict with any certainty what number it will land on, but if I roll sixty times, I can expect it to land on 1 ten times, 2 ten times, 3 ten times, etc.

The probability of an outcome is the proportion of times the outcome would occur for a large number of repetitions.

Example: The probability of a die landing on 4 is the proportion of times a die lands on 4 for a large number of repetitions.

The set of all possible outcomes of an event is the sample space of the event.

Example: For the event "roll a die and observe what number it lands on" the sample space contains all possible numbers the die could land on.

S = { 1, 2, 3, 4, 5, 6 }

An event is an outcome (or a set of outcomes) from a sample space.

Example 1: When flipping three coins, an event may be getting the result THT. In this case, the event is one outcome from the sample space.

Example 2: When flipping three coins, an event may be getting two tails. In this case, the event is a set of outcomes (HTT, THT, TTH) from the sample space.

An event is usually denoted by a capital letter. For example, call getting two tails event A.

The probability of event A is denoted P(A).

Probability Rules:

The probability of any event is between 0 and 1. A probability of 0 indicates the event will never occur, and a probability of 1 indicates the event will always occur.

$$0 \le P(A) \le 1$$

If S is the sample space, then P(S) = 1. Some outcome in the sample space will occur.

The probability that event A does not occur is one minus the probability that A does occur. That A will not occur is called the complement of A and is denoted A^c.

$$P(A^c) = 1 - P(A)$$

Example: When flipping two coins, the probability of getting two heads is 0.25. The probability of not getting two heads is 0.75.

If events A and B are disjoint (they have no outcomes in common), then the probability that A or B occurs is the probability that A occurs ~~or~~ plus the probability that B occurs.

$$P(A \text{ or } B) = P(A) + P(B)$$

Example: Let event A be rolling a die and landing on an even number, and event B be rolling a die and landing on an odd number.

The outcomes for A are {2, 4, 6} and the outcomes for B are {1, 3, 5}. These events are disjoint because they have no outcomes in common.

So the probability of A or B (landing on either an even or an odd number) equals the probability of A plus the probability of B.

$$P(A \text{ or } B) = P(A) + P(B) = \frac{3}{6} + \frac{3}{6} = 1$$

Events A and B are independent if knowing that one occurs does not change the probability that the other occurs.

Example: Roll a yellow die and a red die. Event A is the yellow die landing on an even number, and event B is the red die landing on an odd number. These two events are independent, because the probability of A does not change the probability of B.

If events A and B are independent, then the probability of A and B equals the probability of A times the probability of B.

$$P(A \text{ and } B) = P(A) * P(B)$$

Example: The probability than the yellow die lands on an even number and the red die lands on an odd number is:

$$P(A \text{ and } B) = P(A) * P(B) = \frac{3}{6} \cdot \frac{3}{6} = \frac{1}{4}$$

If events A and B are independent, then their complements, A^c and B^c are also independent, and A^c is independent of B.

The probability that event A occurs if we know for certain that event B will occur is called conditional probability.

The conditional probability of A given B is denoted: $P(A|B)$

If events A and B are independent then knowing that event B will occur does not change the probability of A so for independent events:

$$P(A|B) = P(A)$$

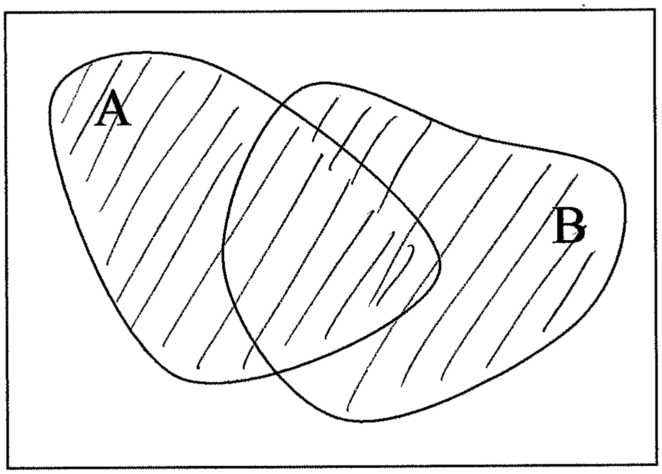
Example: When flipping a coin twice, what is the probability of getting heads on the second flip if the first flip was a head?

Event A: getting heads on first flip
Event B: getting heads on second flip

Events A and B are independent since the outcome of the first flip does not change the probability of the second flip, so...

$$P(B|A) = P(B) = \frac{1}{2}$$

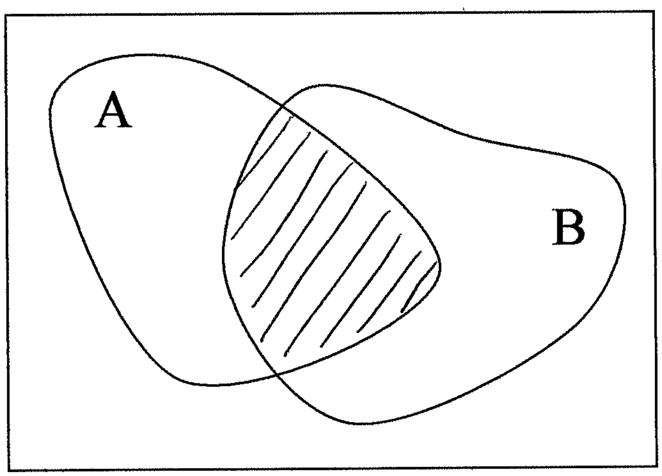
The union of two or more events is the event that at least one of those events occurs.



Addition Rule for the Union of Two Events:

$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \text{ and } B)$

The intersection of two or more events is the event that all of those events occur.



Multiplication ~~Addition~~ Rule for the Intersection of Two Events:

$P(A \text{ and } B) = P(A) * P(B|A)$

$P(B|A) = \dfrac{P(B \cap A)}{P(A)}$ $\qquad$ $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

Example: In our class of 36 students, we found that 25 students like to listen to rap music, 22 students like to listen to alternative music, and 16 students like to listen to both rap and alternative music.

Find the probabilities of the following events:

A: a student likes to listen to rap 25/36

B: a student likes to listen to alternative 22/36

A or B: a student likes to listen to rap **or** alternative 31/36

A and B: a student likes to listen to rap **and** alternative 16/36

Describe the following events:

A^c: Students who don't like rap $P(A^c) = 11/36$

B^c: Students who don't like alternative $P(B^c) = 14/36$

A^c or B^c: don't like rap or don't like alternative $P(A^c \text{ or } B^c) = 20/36$

A^c and B^c: don't like rap and don't like alternative $P(A^c \text{ and } B^c) = 5/36$