

1. Consider a standard deck of playing cards.

a. One card is drawn. What is the probability it is an ace or red?

$$\begin{aligned} P(\text{ace or red}) &= P(\text{ace}) + P(\text{red}) - P(\text{ace and red}) \\ &= \frac{4}{52} + \frac{26}{52} - \frac{2}{52} \\ &= \frac{28}{52} \end{aligned}$$

b. Two cards are drawn without replacement. What is the probability they are both aces?

$$P(\text{two aces}) = \frac{4}{52} \cdot \frac{3}{51} = \frac{12}{2652}$$

c. What is the probability of getting 5 hearts in a row?

$$P(5 \text{ hearts}) = \frac{13}{52} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \cdot \frac{9}{48} \approx 0.0005$$

d. I draw one card and look at it. I tell you it is red.

i. What is the probability it is a heart?

$$P(\text{heart} | \text{red}) = \frac{P(\text{heart and red})}{P(\text{red})} = \frac{\frac{13}{52}}{\frac{26}{52}} = \frac{13}{26} = \frac{1}{2}$$

ii. And what is the probability it is red, given that it is a heart?

$$P(\text{red} | \text{heart}) = \frac{P(\text{red and heart})}{P(\text{heart})} = \frac{\frac{13}{52}}{\frac{13}{52}} = 1$$

e. Are “red card” and “spade” independent? Mutually exclusive (disjoint)?

$$P(\text{red} | \text{spade}) = 0$$

$$P(\text{red}) = \frac{1}{2}$$

∴ not independent

they are disjoint since there are no red spades

f. Are “red card” and “ace” independent? Mutually exclusive (disjoint)?

$$P(\text{red card} | \text{ace}) = \frac{1}{2}$$

$$P(\text{red}) = \frac{1}{2}$$

∴ independent

not disjoint

g. Are “face card” and “king” independent? Mutually exclusive (disjoint)?

$$P(\text{face} | \text{king}) = 1$$

$$P(\text{face}) = \frac{12}{52}$$

∴ not independent

not disjoint

2. Use the following table to answer the questions below:

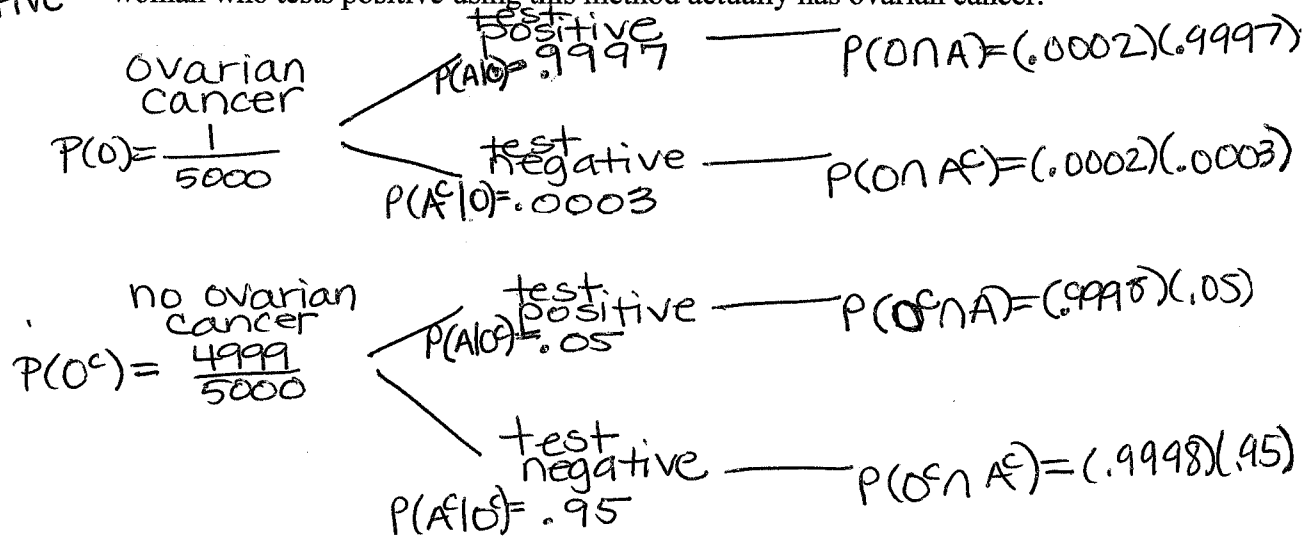
	Jeans	Other	Total
Male			
Female			
Total			

- What is the probability that a male wears jeans?
- What is the probability that someone wearing jeans is male?
- Are being male and wearing jeans disjoint?
- Are gender and attire independent?

3. In April 2003, *Science* magazine reported on a new computer-based test for ovarian cancer, "clinical proteomics," that examines a blood sample for the presence of certain patterns of proteins. Ovarian cancer, though dangerous, is very rare, afflicting only 1 of every 5000 women. The test is highly sensitive, able to correctly detect the presence of ovarian cancer in 99.97% of women who have the disease. However, it is unlikely to be used as a screening test in the general population because the test gave false positives 5% of the time. Why are false positives such a big problem? Draw a tree diagram and determine the probability that a woman who tests positive using this method actually has ovarian cancer.

Event O:
ovarian
cancer

Event A:
test
positive



$$P(O|A) = \frac{P(O \cap A)}{P(A)} = \frac{(0.0002)(0.9997)}{(0.0002)(0.9997) + (0.9998)(0.05)} \approx 0.00398$$