Chapter 16: Random Variables

Discrete and Continuous Random Variables:

A variable is a quantity whose value changes. A discrete variable is a variable whose value is obtained by counting. A discrete variable does not take on all possible values within a given interval.

Examples:
number of students present
number of red marbles in a jar
number of heads when flipping three coins

A continuous variable is a variable whose value is obtained by measuring
A continuous variable takes on all possible values within a given interval.

Examples:
height of students in class
time it takes to get to school
distance traveled between classes

A random variable is a variable whose value is a numerical outcome of a random phenomenon.

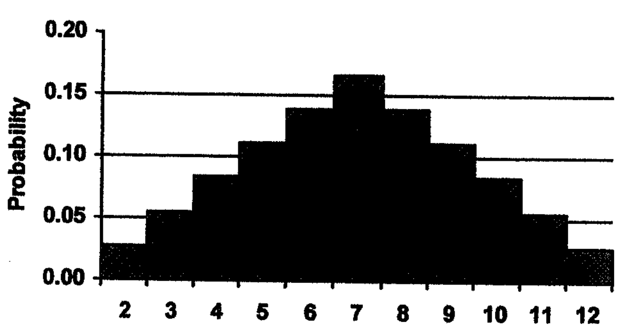
- A random variable is denoted with a capital letter. A particular value of a random variable will be denoted with a lower case letter.
- The probability distribution of a random variable *X* tells what the possible values of *X* are and how probabilities are assigned to those values.
- A random variable can be discrete or continuous

A discrete random variable *X* has a countable number of possible values.

Example: Let *X* represent the sum of two dice. Then the probability distribution of *X* is as follows:

X	2	3	4	5	6	7	8	9	10	11	12
P(*X*)	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

To graph the probability distribution of a discrete random variable, construct a histogram. The probability distribution for the sum of two dice is given by:



A continuous random variable X takes all values in a given interval of numbers.

- The probability distribution of a continuous random variable is shown by a density curve. The area under a density curve (no matter what shape it has) is 1.
- The probability that X is between an interval of numbers is the area under the density curve between the interval endpoints
- The probability that a continuous random variable X is exactly equal to a number is zero

Means and Variances of Random Variables:

The mean of a random variable X is called the expected value of X. The mean of a discrete random variable, X, is its weighted average. Each value of X is weighted by its probability. To find the mean of X, multiply each value of X by its probability, then add all the products.

$$E(X) = x_1p_1 + x_2p_2 + \cdots + x_kp_k$$
$$= \sum x_ip_i$$

Law of Large Numbers:

As the number of observations increases, the mean of the observed values, $\bar{x}$, approaches the mean of the population, μ.

The more variability in the outcomes, the more trials are needed to ensure $\bar{x}$ is close to μ.

Rules for Means:

If X is a random variable and *a* and *b* are fixed numbers, then

$E(a+bX) = a + b\,E(X)$

$\mu_{a+bX} = a + b\mu_X$

*NOTE: $E(X+X) \neq E(2X)$

$E(X_1 + X_2) = E(X_1) + E(X_2)$

If X and Y are random variables, then

$E(X+Y) = E(X) + E(Y)$

$\mu_{X+Y} = \mu_X + \mu_Y$

Example:

Suppose the equation Y = 20 + 10X converts a PSAT math score, X, into an SAT math score, Y. Suppose the average PSAT math score is 48. What is the average SAT math score?

$E(Y) = 20 + 10\,E(X)$

$= 20 + 10(48) = 500$

Example:

Let $\mu_X = 625$ represent the average SAT math score.

Let $\mu_Y = 590$ represent the average SAT verbal score.

$E(X+Y)$ represents the average combined SAT score. So the average combined total SAT score is:

$E(X+Y) = E(X) + E(Y) = 625 + 590 = 1215$

The Variance of a Discrete Random Variable:

If X is a discrete random variable with mean μ, then the variance of X is

$$Var(X) = (x_1 - \mu_X)^2 p_1 + (x_2 - \mu_X)^2 p_2 + \cdots + (x_k - \mu_X)^2 p_k +$$
$$= \sum (x_i - \mu_X)^2 p_i$$

The standard deviation σ_X is the square root of the variance.

$$SD(X) = \sqrt{Var(X)} = \sqrt{\sum (x_i - \mu_X)^2 p_i}$$

Rules for Variances:

If X is a random variable and *a* and *b* are fixed numbers, then

$$VAR(a+bX) = b^2 VAR(X)$$
$$\sigma^2_{a+bX} = b^2 \sigma^2_X$$

If X and Y are independent random variables, then

$$VAR(X+Y) = VAR(X) + VAR(Y)$$
$$VAR(X-Y) = VAR(X) - VAR(Y)$$
$$\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y$$

Example:
Suppose the equation Y = 20 + 10X converts a PSAT math score, X, into an SAT math score, Y. Suppose the standard deviation for the PSAT math score is 1.5 points. What is the standard deviation for the SAT math score?

$$VAR(20+10X) = 100\,VAR(X) = 100(1.5)^2 = 225$$
$$SD(20+10X) = \sqrt{225} = 15$$

Suppose the standard deviation for the SAT math score is 150 points, and the standard deviation for the SAT verbal score is 165 points. What is the standard deviation for the combined SAT score?

*** Because the SAT math score and SAT verbal score are not independent the rule for adding variances does not apply!