Chapter 20: Testing Hypotheses About Proportions

Suppose we combine four decks of cards, and we suspect that they have been tampered with, such that the proportion of red cards is less than 0.5.

Can we prove it if we look at less than half the deck? NO

Can we convince ourselves if we look at less than half the deck? YES

- Draw a sample of two cards. Suppose they are both black. Are you convinced that the true proportion of red cards is less than 0.5? NO
- Draw a sample of 20 cards. Suppose 8 are ~~black~~ red. Are you convinced that the true proportion of red cards is less than 0.5?

We still don't know for sure, but we can quantify our uncertainty by finding out how unusual the above result would be if the true proportion of red cards were in fact 0.5.

We will conduct a hypothesis test in the following way:

1. Define the parameter.
   p: proportion of red cards in the deck
2. State the null hypothesis and the alternative hypothesis

$H_0: p = 0.5$

$H_A: p < 0.5$

3. Check the necessary conditions and assumptions for constructing a Normal model.

① random

② $np_0 = 52(0.5) = 26 \geq 10$

$nq_0 = 52(0.5) = 26 \geq 10$

③ $N \geq 200$ cards (in four decks)

Since there are at least 10 ~~successes~~ expected and 10 ~~failures~~ expected, the population of all cards in the deck is at least 200, and the data came from a random sample, the sampling model for $\hat{p}$ is approximately

$$N\left(0.5, \sqrt{\frac{0.5(0.5)}{20}}\right)$$

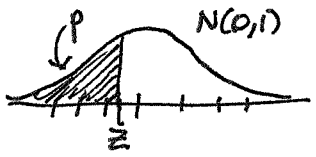
$N(0.5, .1118)$

4. Name the procedure.
   A one-proportion z-test may be used.

5. Calculate the test statistic.

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.40 - 0.50}{\sqrt{\frac{(.50)(.50)}{20}}} \approx -0.894$$

6. Obtain the p-value.



$p = P(\hat{p} < 0.4) = P(z < -0.894)$
$= \text{normalcdf}(-1E99, -0.894)$
≈ 0.1857

7. Make a decision regarding the null hypothesis.
   Since the p-value is not unusually small we fail to reject the null hypothesis.

8. State your conclusion in context.
   If the true proportion of red cards (in a deck of 208) were 0.5, we would expect to see a sample proportion at least as extreme as the one we observed (0.4) in about 19 out of every 100 samples of this size. This is not strong enough evidence to conclude that the true proportion of red cards is less than 0.50.

Remember **PHANTOMS**:

P arameter
H ypotheses
A ssumptions
N ame the test
T est statistic
O btain p-value
M ake decision
S tate conclusion in context