Chapter 21: Inference as Design

If we use the results of a significance test to make a decision, then we either reject the null hypothesis in favor of the alternative hypothesis, or we accept the null hypothesis.

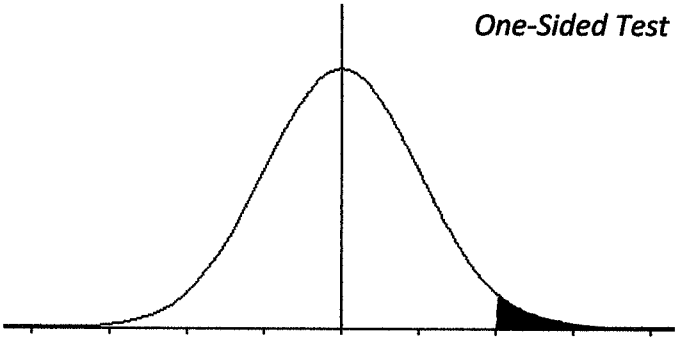
We hope that our decision will be correct, but it is possible that we make the wrong decision. There are two ways to make a wrong decision:

- We can reject the null hypothesis when in fact it is true. This is called a Type I Error.
- We can accept (fail to reject) the null hypothesis when in fact it is false. This is called a Type II Error.

		Truth about the population	
		H_0 is TRUE	**H_0 is FALSE**
Decision based on sample	reject H_0	Type I Error $p = \alpha$	★ Correct Decision $p = 1 - \beta$ POWER
	fail to reject H_0	Correct Decision	Type II Error $p = \beta$

We are interested in knowing the probability of making a Type I Error and the probability of making a Type II Error.

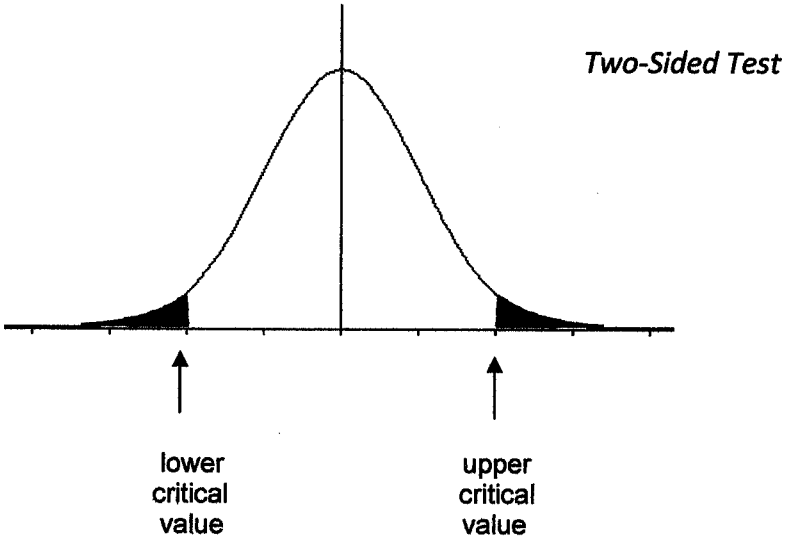
A Type I Error occurs if we reject the null hypothesis when it is in fact true.
When do we reject the null hypothesis? When we assume that it is true and find that the statistic of interest falls within the rejection region. The probability that the statistic falls in the rejection region is the area of the shaded region, or α.



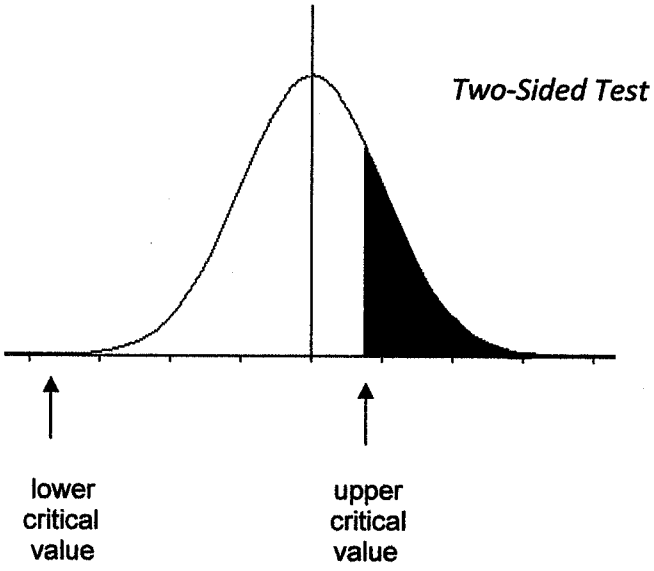
Therefore the probability of a Type I Error is equal to the significance level α of a fixed level test. The probability that the test will reject the null hypothesis H_0 when in fact H_0 is true is α.

A Type II Error occurs if we fail to reject the null hypothesis when it is in fact false. **When do we accept (or fail to reject) the null hypothesis?** When we assume that it is true and find that the statistic of interest falls outside the rejection region.

However, the probability that the statistic falls outside the rejection region is NOT the area of the unshaded region. *Think about it...* If the null hypothesis is in fact false, then the picture is NOT CORRECT... it is off center.



To calculate the probability of a Type II Error, we must find the probability that the statistic falls outside the rejection region (the unshaded area) given that the mean is some other specified value.



The probability of a Type II Error tells us the probability of failing to reject the null hypothesis when it is actually false. The complement of this would be the probability of **not accepting** (in other words *rejecting*) the null hypothesis when it is actually false. To calculate the probability of rejecting the null hypothesis when it is actually false, compute 1 – P(Type II Error). This is called the power of a significance test.